647510 **BSCbooCC5030** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
March/April-2023(Old Course)
BOOLEAN ALGEBRA AND COMPLEX ANALYSIS-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer any one **[7]**

(1) In usual notation prove that $(a * b) * c = a * (b * c)$ and $(a \oplus b) \oplus c = a \oplus (b \oplus c)$

(2) State and prove Distributive inequalities.

Q.1(B) Answer any one **[4]**

(1) In usual notation show that (S_{30}, D) is poset.

(2) Let $(L, \leq)$ be a Lattice and $a, b \in L$, then prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

Q.1(C) Answer any one **[3]**

(1) In usual notation prove that $xRy \Leftrightarrow [x] = [y]$

(2) Define cover, Draw Hasse diagram of $(P(A), \subseteq)$ for $A = \{a, b, c\}$

Q.2(A) Answer any one **[7]**

(1) If $(B, *, \oplus, ', 0, 1)$ is Boolean algebra and $x_1, x_2 \in B$ then prove that $A(x_1 * x_2) = A(x_1) \cap A(x_2)$.

(2) Obtain sum of product canonical form of $\alpha(x_1, x_2, x_3) = x_2 \oplus x_3$

Q.2 (B) Answer any one **[4]**

(1) For Boolean statement prove that
$(a' * b' * c) \oplus (a * b' * c) \oplus (a * b' * c') = b' * (a \oplus c)$

(2) For a Boolean algebra B, prove that $a \neq 0$ is an atom in B if and only if for any $x \in B$, $a * x = 0$ or $a * x = a$.

Q.2(C) Answer any one **[3]**

(1) Define Direct product of two Boolean algebra.

(2) Define Minterm and Maxterm in Boolean algebra.

Q.3(A) Answer any one **[7]**

(1) In usual notation obtain Cauchy – Riemann conditions in polar form.

(2) Find an analytic function $f(z) = v + i v$ such that $u - v = e^{-x}(\sin y + \cos y)$.

Q.3(B) Answer any one **[4]**

(1) Prove that $u = \log r$ is a harmonic function and find it's conjugate.

(2) Obtain Laplace equation in polar form.

Q.3(C) Answer any one **[3]**

(1) Prove that $\sin z$ is an analytic function.

(2) Show that z^2 is entire function.

Q.4(A) Answer any one **[7]**

(1) State and prove Cauchy's integral formula.

(2) Find $\int_C \frac{dz}{z-\frac{2}{z}}$; where C: $|z-1| = 2$.

Q.4(B) Answer any one **[4]**

(1) In usual notation prove that $\left|\int_c f(z)dz\right| \leq$ ML.

(2) Find: $\int_c \frac{e^z}{(z-4)(z-2)}$ dz; where C: | z | = 3.

Q.4(C) Answer any one **[3]**

(1) Find $\int_c \frac{z}{(z-1)}$ dz; where C: $|z|$ = 2.

(2) Find: $\int_c \frac{dz}{(z-3)}$ where C: | z - 2| = 5.

Q.5(A) Answer any one **[7]**

(1) State and prove Liouville's theorem.

(2) State and prove Morera's theorem.

Q.5(B) Answer any one **[4]**

(1) Find: $\int_c \frac{z^2+3}{z^2(z-4)}$ dz where C: |z| = 1

(2) In usual notation state and prove Cauchy's inequality.

Q.5(C) Answer any one **[3]**

(1) If C: |z| = 3 is the closed contour and the integral is fallen in positive sense and g(z) = $\int_c \frac{2s^2-s-2}{s-z}$ ds then prove that g(2) = 8πi.

(2) Find: $\int_c \frac{\cosh z\, dz}{z^3}$ where C: | z | = 1.

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